

# C. U. SHAH UNIVERSITY

## Summer Examination-2020

**Subject Name : Engineering Mathematics - I**

**Subject Code : 4TE01EMT3**

**Branch: B. Tech (All)**

**Semester : 1**

**Date : 26/02/2020**

**Time : 02:30 To 05:30**

**Marks : 70**

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1                      Attempt the following questions:                      (14)**

- a)** If  $y = \sin 2x \cos 2x$  then  $y_n$  equal to  
 (A)  $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$     (B)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$   
 (C)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$     (D) none of these
- b)** If  $y = \frac{3x+2}{2x+3}$ , then  $y_n$  equal to  
 (A)  $\frac{5(-1)^{n+1} n! 2^{n-1}}{(2x+3)^{n+1}}$     (B)  $\frac{5(-1)^n n! 2^n}{(2x+3)^{n+1}}$     (C)  $\frac{5(-1)^n n! 2^n}{(2x+3)^{n-1}}$   
 (D) none of these
- c)** If  $y = \log(1+x)$ , then  $x$  equal to  
 (A)  $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!}$     (B)  $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$   
 (C)  $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \frac{y^4}{4!} + \dots$     (D) none of these
- d)**  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \infty$  represent expansion of  
 (A)  $\sinh x$     (B)  $\cosh x$     (C)  $\cos x$     (D)  $e^x$
- e)**  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \underline{\hspace{2cm}}$   
 (A)  $a-b$     (B)  $\log(a-b)$     (C)  $\log ab$     (D)  $\log\left(\frac{a}{b}\right)$
- f)**  $\lim_{x \rightarrow \infty} \frac{x^n}{e^{ax}}$  ( $n$  being a positive integer and  $a > 0$ ) =  $\underline{\hspace{2cm}}$



- (A) -1 (B) 0 (C) 1 (D) None of these
- g) If  $P = r \tan \theta$ , then  $\frac{\partial P}{\partial r}$  is equal to  
 (A)  $\sec^2 \theta$  (B)  $\tan \theta$  (C)  $\tan \theta + r \sec^2 \theta$  (D)  $\frac{1}{2} \tan \theta$
- h) If  $f(x, y) = 0$ , then  $\frac{dy}{dx}$  is equal to  
 (A)  $\frac{\partial f / \partial x}{\partial f / \partial y}$  (B)  $\frac{\partial f / \partial y}{\partial f / \partial x}$  (C)  $-\frac{\partial f / \partial y}{\partial f / \partial x}$  (D)  $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- i) If  $u(x, y, z) = 0$  then the value of  $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$  is equal to  
 (A) 1 (B) -1 (C) 0 (D) none of these
- j) If  $f_1 = \frac{vw}{u}$ ,  $f_2 = \frac{wu}{v}$ ,  $f_3 = \frac{uv}{w}$ ; then  $\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$  is equal to  
 (A) 0 (B) 1 (C) 3 (D) none of these
- k) The value of  $\sum_{k=1}^{10} \left( \sin \frac{2\pi k}{11} - i \cos \frac{2\pi k}{11} \right)$  is  
 (A) -1 (B) 0 (C) -i (D) i
- l) If  $\left( \frac{1-i}{1+i} \right)^{100} = a + ib$ , then  
 (A)  $a = 2, b = -1$  (B)  $a = 1, b = 0$  (C)  $a = 0, b = 1$  (D)  $a = -1, b = 2$
- m) If  $A$  is a non-zero column vector  $(n \times 1)$ , then the rank of matrix  $AA^T$  is  
 (A) 0 (B) 1 (C)  $n-1$  (D)  $n$
- n) An eigenvalue of a square matrix  $A$  is  $\lambda = 0$ . Then  
 (A)  $|A| \neq 0$  (B)  $A$  is symmetric (C)  $A$  is singular  
 (D)  $A$  is skew-symmetric

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) If  $y = \tan^{-1} x$  then find the value of  $y_n(0)$ . (5)

b) Expand  $f(x) = \frac{e^x}{e^x + 1}$  in powers of  $x$  up to  $x^3$  by Maclaurin's series. (5)

c) If  $V = \frac{1}{r}$  where  $r^2 = x^2 + y^2 + z^2$  then show that  $V(x, y, z)$  satisfies (4)

$$\text{Laplace's equation } \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

**Q-3 Attempt all questions (14)**

a) If  $y = a \cos(\log x) + b \sin(\log x)$  then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0.$$

b) Prove that  $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$  (5)



c) Evaluate:  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\log x} \right)$ . (4)

**Q-4 Attempt all questions** (14)

a) Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$  (5)

b) If  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \theta$  then find  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . (5)

c) Expand  $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$  in powers of  $(x-3)$ . (4)

**Q-5 Attempt all questions** (14)

a) If  $u = \tan^{-1} \left( \frac{x^2 + y^2}{x-y} \right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$ . (5)

b) Evaluate:  $\lim_{x \rightarrow 0} \frac{a}{x^2} \left[ \frac{\sin kx}{\sin lx} - \frac{k}{l} \right]$  (5)

c) If  $y = \frac{x^4}{(x-1)(x-2)}$  then find  $y_n$ . (4)

**Q-6 Attempt all questions** (14)

a) Using the formula  $R = \frac{E}{I}$ , find the maximum error and percentage of error in R if  $I = 20$  with a possible error of 0.1 and  $E = 120$  with a possible error of 0.05 and  $R = 6$ . (5)

b) Expand  $\sin^5 \theta \cos^2 \theta$  in a series of sines of multiples of  $\theta$ . (5)

c) Find the inverse of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$  by Gauss-Jordan reduction method. (4)

**Q-7 Attempt all questions** (14)

a) Find the eigenvalues and eigenvectors of the matrix  $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ . (5)

b) Find the continued product of all the values of  $\left( \frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{\frac{3}{4}}$ . (5)

c) If  $\tan(\alpha + i\beta) = x + iy$  then prove that  $x^2 + y^2 + 2x \cot 2\alpha = 1$ . (4)

**Q-8 Attempt all questions** (14)

a) Investigate for what values of  $\lambda$  and  $\mu$  the equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$ , have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (5)

b) Find the fourth roots of unity and sketch them on the unit circle. (5)

c) Verify Caley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ . (4)

